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Basic Principles of Fresnel Antenna Arrays

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*Dedicated to the father, friend and colleague, all in one,
and equally to our mother. Without their help and support,
this book would never have been written.*

Foreword

The interest in Fresnel antennas has been growing steadily since the late 1980s. The simple but ingenious Fresnel zone concept has been extended greatly and exploited to create different types of Fresnel zone antennas including the offset Fresnel zone antennas, the phase correcting Fresnel lenses and zone plate antennas, and reflect arrays. This is primarily driven by the increasing demand on low-cost, low-profile, and high-gain antennas.

I know Professor I. Mimin and Professor O Mimin “over the net” since early 1990s, and I have always been impressed by their untiring effort to pursue various new ideas related to Fresnel zone diffraction and Fresnel zone antennas. It is encouraging to see the publication of their new book, “*Basic Principles of Fresnel Antenna Array*,” which covers the research on Fresnel antenna arrays. For a given antenna aperture, employing an array of Fresnel antennas as opposed to a single one can reduce the distance between the feed and the aperture, thereby reducing the profile of the overall system. On the other hand, a feed network is required to connect all the feeds, and the losses in the feed network need to be managed. It is expected the book will attract more interest in Fresnel zone antennas and stimulate new ideas on this fascinating subject.

Dr. Y. Jay Guo
Clayton South, VIC

Foreword

This work is an important addition to the field of array antennas. It also extends the knowledge and applications of Fresnel zone plate lens antennas (FZPLAs) and is the first study of FZPLAs in arrays. Profs. Igor and Oleg Minin have done extensive research and development on FZPLAs. They are among the world's most famous researchers of the Fresnel zone plate antennas, having written four books (two in Russian) and numerous published articles. The Drs. Minin have made extensive millimeter-wave measurements on Fresnel zone plate antennas and rigorous comparisons with theory. This book is an excellent summary of the state of the art of diffractive optical elements (DOE) and introduces the application to array antennas. The book is very comprehensive and thorough, and offers innovative new ideas.

Most of their work is applicable to zone plates having comparable diameters and focal lengths, the so-called large-angle zone plate, which is the configuration most often used at microwave, millimeter-wave, and terahertz frequencies. This book addresses the need for low-cost, low-profile, and lightweight antennas. A new hexagonal FZPL antenna is described. This has the advantage over the usual circular cross-section FZPLA in that its shape fits more compactly into an array. A technique involving Fresnel rotation is described. This improves the radiation pattern of the hexagonal FZPL antenna. In addition, the concept of changing the reference phase of the Fresnel zone radii from 0° to a more optimal number up to 180° is described. These last two ideas improve the radiation characteristics.

The technology of diffractive optics in the microwave to terahertz frequency range has seen much activity in recent years. This is the fifth book to be published on FZPL antennas, and in the past two decades some 100 research articles have appeared in the worldwide literature. The field of diffractive optical elements has been used in many applications, including radar, radiometry, missile terminal guidance seekers, point-to-point communications, and field tests of atmospheric effects, especially in cases where low cost, low attenuation, low weight, low physical volume, and ease of manufacturing are the considerations.

This book begins with a review of elementary basics of antennas, defining such quantities as radiation power density and intensity, directivity, effective area of an antenna, and radiation pattern, parameters relevant to any antenna. Subjects relating to arrays are also described, including such topics as array factor, diversity combining, and grating lobe criteria. Fresnel zone plate antennas are also reviewed, comparing lenses and zone plates and giving Fresnel zone plate antenna design information. This includes the information about reflector-backed FZPLAs and reconfigurable zone plates utilizing photoconductive material or mechanical shutters. One chapter considers FZPLA candidates for arrays based on geometry:

rectangular, square, circular, polygonal, hexagonal, star-shaped, and arbitrary-shaped apertures. Perforated dielectric Fresnel lenses are considered in another chapter.

A final chapter treats arrays of small lenses, lens arrays based on Luneberg lenses, fly's-eye imaging concepts, waveguide lens array technology, and several other unusual applications. The net result is an extremely diverse coverage of numerous configurations and applications. The book should see broad use and wide application. Profs. Igor and Oleg Minin have again produced a significant document that will be valuable to antenna designers and those designing systems such as radar or communications.

Dr. James C. Wiltse
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Introduction

Wireless and satellite communications are vital in the daily activities of the average individual and business. Most services can now be received from anywhere at anytime. These services must deliver wireless high bandwidth multimedia signals which provide reliably simultaneous access to voice, fax, high speed internet, and video. Also homeland security, nondestructive testing, imaging technologies are actually today. Antennas represent a critical technology in any of these wireless systems.

Achieving a lowcost, lowprofile, and lightweight antenna in the microwave/millimeter wave band (MMW) is not trivial. The core problem is generally that low aperture efficiencies result with existing lowprofile technologies. The parabolic reflector, reflectarray, and dielectric hyperbolic lens are all high gain, high efficiency antennas with desirable radiation characteristics but they are not lowprofile. Each one consists of a feed placed in front of the aperture by a certain distance which causes the antenna depth to be large even at MMW. There are also stringent tolerance issues that exist in each of their construction which make them costly to fabricate at these high frequencies. Fresnel lens antennas, including Fresnel zone plates [15], are also not very low profile due to their feed being placed at the focal point in front of the aperture in a similar manner to the other high gain antennas.

The most common low profile technology is the microstrip antenna, which is typically used in an array to achieve higher gain. At higher frequencies, such as those in the MMW, the microstrip array feed network is very lossy which seriously degrades the aperture efficiency. The slotted waveguide array is another potential low profile antenna option, but its cost is prohibitive owing to the tight fabrication tolerances required at these frequencies. The only other alternative low profile antenna is the dielectric grating but, similar to the slotted waveguide array, fabrication becomes more complex and costly in the MMW.

A potential solution to the low profile antenna problem at MMW lies in using lens technology in an array. By replacing a large diameter lens with an array of smaller lenses, the overall profile of the lens can be significantly reduced. However, there are some drawbacks to lens arrays such as the requirement for more feeds and the degradation of sidelobe performance. Very little work has been performed on using lenses in arrays though, which means that this area remains relatively unexplored.

The Fresnel zone plate lens (FZPL) antenna, in particular, is an interesting candidate for the array element since it is the lowest cost, lowest profile, and

lightest weight lens antenna [15]. The aperture of the FZPL antenna has the lowest profile of the Fresnel lens family of antennas because it is simply printed on a thin substrate which does not need to be of microwave grade. This also makes the FZPL antenna inexpensive to fabricate.

It could be noted that a Fresnel zone plate works by interference and a Fresnel lens by refraction. The focal length of a Fresnel lens is not proportional to the wavelength of radiation, but for a zone plate, it is.

It is interesting also to note that in the paper on optical processing for synthetic aperture array radar, Cutrona et al. [6] point out that the synthetic aperture recorded signals are one dimensional Fresnel zone-plate lenses. In the paper [7] a method of electronically processing synthetic aperture arrays utilizing the Fresnel zone-plate lens concept was also presented.

Different from the conventional lens which uses curved surfaces to refract radiation, so-called a digital lens is made of a transparent flat plate with numerous micro zones, each zone has its location and optical characteristics precisely and in common case digitally defined. When radiation passes through these zones, the light waves *interfere* with each other due to phase differences and results in light wave manipulation. This manipulation technology can be used to deliver superior quality of images and it is widely applicable to many other industries such as radar *antenna* and *communications*.

In general, the term diffractive optical elements (DOE) refer to those that are based on the utilization of the wave nature of radiation. The DOEs in common case are based on grating composition. The grating effect is dominant and defines their function and limitations. In general, grating dispersion is much stronger than prism dispersion. Thus, frequency properties (or chromatic dispersion) strongly influences to the imaging properties of DOEs. The relative conditional categorization of DOEs can be divided into several main subsections: diffractive lenses (elements that perform functions similar to conventional refractive lenses, e.g. they form images); diffractive antennas (quasi-optical elements that form arbitrary beampattern in far field); kinoforms (DOE whose phase modulation is introduced by a surface relief structure); binary optics; diffractive phase elements (DOEs that introduce phase change). The DOE can be seen either also as an aperture synthesis array. Beams from the individual apertures are recombined by diffraction and interference. The apertures (void circular, rectangles, hexagonal or more complex shapes) are positioned so that at the first order of diffraction (2π phase shift from one aperture subset to the next), an incoming plane wave is turned into a spherical outgoing wavefront.

One of the factors that have stimulated much of the recent interest in diffractive optics at any frequency waveband has been the increased optical performance of such optical elements. This allows the fabrication of optical elements that are smaller (compared to wavelength), lighter and cheaper to fabricate, are more rugged and have superior performance that the conventional optical or/and quasi-optical components they often replace. Important, the design capabilities for

binary optics now available can make possible the design and manufacture to components including antennas having optical and focusing properties never before produced.

FZPL antenna is one of the simple digital lenses or DOE. Flat surfaces are two dimensional, therefore much cheaper to fabricate than three dimensional contour surfaces. It's also more precise due to digitally defined microscopic zones and simple geometric shape. With the conventional technology, it's very difficult, if not possible to produce a lens array on a single sheet of material, not to mention the cost. To further lower the fabrication cost, lenses even can be *printed* on a flat surface.

The traditional circular FZPL antenna, however, does not perform well in an array. This is because the elements cannot be placed any closer together than when their edges touch at a single point, which does not provide an adequate separation to minimize grating lobes [8]. As such, a novel hexagonal FZPL (HFZPL) antenna [9] is described in details which can be more effectively packed in an array due to its shape.

Prior to considering the HFZPL antenna in an array, two main ideas described as methods to potentially improve the radiation characteristics. The first idea is to change the reference phase of the Fresnel zone radii from the standard 0° to a more optimal value between 0 and 180 [10]. To further improve the radiation characteristics of the HFZPL antenna, a technique involving Fresnel zone rotation [9] are also described. This method might also provide interesting beamshaping options when used on the elements in an array.

The book is intended to serve engineers, researchers and student in the field of antennas, microwave/millimeter wave/THz wave engineering and telecommunications. Also the authors believe that the book will also be of interest to designers of optical systems because, with scaling effects taken into account, the characteristics of diffractive quasioptical elements are valid for diffractive focusing elements of integrated optics.

We are greatly indebted to Prof. Jim Wight (Chancellor's University), Dr. Aldo Petosa (CRC, Canada) and specially to Dr. Sara M. Stout-Grandy, the big number of results described in the book were obtained during her 3 years Ph.D. thesis work titled "Investigation of Planar Fresnel Zone Plate Antennas" under the supervision of Dr.A.Petosa, Prof I. Minin and Prof. J. Wight, for permission to use figures and some of joint results.

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Chapter 1

The Brief Elementary Basics of Antenna Arrays

For some applications, single-element antennas are unable to meet the gain or radiation pattern requirements. Combining several single antenna elements in an array can be a possible solution.

1.1 Some Basic Antenna Parameters Definitions

1.1.1 Radiation Power Density

Radiation Power density, W , gives a measure of the average power radiated by the antenna in a particular direction and is obtained by time-averaging the Poynting vector.

$$W_r(r, \theta, \phi) = \frac{1}{2} \operatorname{Re}[E \times H^*] = \frac{1}{2\eta} |\bar{E}(r, \theta, \phi)|^2 \text{ (Watts/m}^2\text{)}, \quad (1.1)$$

where, E is the electric field intensity, H is the magnetic field intensity, and η is the intrinsic impedance (Fig. 1.1).

1.1.2 Radiation Intensity

Radiation intensity, U , in a given direction is the power radiated by the antenna per unit solid angle. It is given by the product of the radiation density and the square of the distance r .

$$U = r^2 W_r \text{ (Watts/unit solid angle)} \quad (1.2)$$

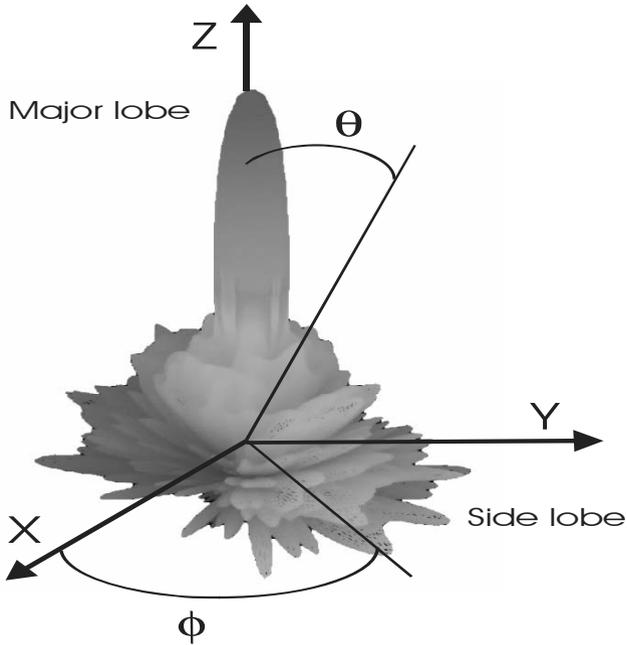


Fig. 1.1 3D polar dimensional radiation power pattern

1.1.3 Total Power Radiated

The total power radiated by the antenna in all the directions is given by:

$$P_{tot} = \int_0^{2\pi} \int_0^{\pi} W_r(r, \theta, \phi) r^2 \sin \theta d\theta d\phi \tag{1.3}$$

$$= \int_0^{2\pi} \int_0^{\pi} U(\theta, \phi) \sin \theta d\theta d\phi \quad (\text{Watts}) \tag{1.4}$$

1.1.4 Directivity

The Directive gain, D_g , is the ratio of the radiation intensity in a given direction to the radiation intensity in all the directions, i.e.:

$$\begin{aligned}
 D_g &= \frac{4\pi U(\theta, \phi)}{P_{tot}} \\
 &= \frac{4\pi r^2 W_r(r, \theta, \phi)}{\int_0^{2\pi} \int_0^\pi W_r(r, \theta, \phi) r^2 \sin(\theta) d\theta d\phi} = \frac{4\pi U(\theta, \phi)}{\int_0^{2\pi} \int_0^\pi U(\theta, \phi) \sin(\theta) d\theta d\phi}
 \end{aligned} \tag{1.5}$$

The Directivity, D_0 , is the maximum value of the directive gain, D_g , for a given direction, i.e.:

$$D_0 = \frac{4\pi U_{\max}(\theta, \phi)}{P_{tot}}, \tag{1.6}$$

where $U_{\max}(\theta, \phi)$ is the maximum radiation intensity.

1.1.5 Effective Area of an Antenna

Consider that an EM wave of power density, P , is intercepted by an antenna of actual physical area, A . The EM wave will induce a voltage, V , across the radiation resistance of the antenna. The antenna can deliver a maximum power, W_{\max} , to a load, $R_L = R_{rad}$, i.e., a load matched to the antenna radiation resistance (assuming ohmic losses are zero). The maximum power that the antenna can deliver to a load is:

$$W_{\max} = \frac{V^2}{2R_{rad}}. \tag{1.7}$$

The effective area, A_e , of an antenna is defined as the area required for gathering enough power from the incident power density, P , of the EM wave in order to deliver power, W_{\max} , to the load.

$$A_e = \frac{W_{\max}}{P}. \tag{1.8}$$

Let us find the effective area of a short dipole. The radiation resistance is

$$R_{rad} = 80\pi^2 \left(\frac{l}{\lambda} \right)^2.$$

The power density at the short dipole is $P = \frac{E^2}{\eta_0} = \frac{E^2}{120\pi}$.

$$\text{From (1.8) } A_e = \frac{V^2}{4PR_{rad}} = \frac{V^2(120\pi)}{4E^2(80\pi^2)} \left(\frac{\lambda}{l}\right)^2 = \frac{3}{8\pi} \left(\frac{V\lambda}{El}\right)^2.$$

Now, the voltage induced across a short dipole is just $V = El$. So the effective area is $A_e = \frac{3}{8\pi} \lambda^2$. The directivity of a short dipole is $D_{max} = 1.5$. Then the relative directivity of a short dipole is:

$$\frac{D_{max}}{A_e} = \frac{1.5}{\left(\frac{3\lambda^2}{8\pi}\right)} = \frac{4\pi}{\lambda^2}.$$

1.1.6 Radiation Pattern

The Radiation Pattern of an antenna can be defined as the variation in field intensity as a function of position or angle. Let us consider an anisotropic radiator, which has stronger radiation in one direction than in another. The radiation pattern of an anisotropic radiator consists of several lobes. One of the lobes has the strongest radiation intensity compared to the other lobes. It is referred to as the Major lobe. All the other lobes with weaker intensity are called Minor Lobes. The width of the main beam is quantified by the Half Power Beamwidth (HPBW), which is the angular separation of the beam between half-power points.

1.2 Antenna Arrays, Radiation Pattern, and Array Factor

Let us consider briefly the basics of antenna arrays following [1, 2]. The antenna elements can be arranged to form a 1, 2 or 3 dimensional antenna array. As a rule in practice either a 1 or 2 dimensional antenna array is used.

The overall radiation pattern changes when several antenna elements are combined in an array. An array consists of two or more antenna elements that are spatially arranged and electrically interconnected to produce a directional radiation pattern. The interconnection between elements, called the feed network, can provide fixed phase to each element or can form a phased array. The geometry of an array and the patterns, orientations, and polarizations of the elements influence the performance of the array. The so-called ‘array factor’ is used: this factor quantifies the effect of combining radiating elements in an array without the element-specific radiation pattern taken into account. The overall radiation pattern of an array is determined by this array factor combined with the radiation pattern of the antenna element. The overall radiation pattern results in a certain directivity and thus gain is linked through the efficiency with the directivity. Directivity and gain are equal if the efficiency is 100%.

1.2.1 Broadside and Fire Arrays

Arrays can be designed to radiate in either broadside, i.e., radiation perpendicular to array orientation (the z-axis) or end fire, i.e., radiation in the same direction as the array orientation (the y-axis). To simplify the problem, we will focus on broadside arrays and only radiation in the z direction is considered. This allows for easy transformation to 2 dimensional planar arrays with the elements in the (xy) plane.

1.2.2 Defining Array Factor

The array factor depends on the number of elements, the element spacing, amplitude and phase of the applied signal to each element. The number of elements and the element spacing determine the surface area of the overall radiating structure. This surface area is called aperture. A larger aperture results in a higher gain. The aperture efficiency quantifies how efficiently the aperture is used. The influence of these parameters will be further explained with the aid of a linear array of isotropic radiating elements. An isotropic radiating element radiates an equal amount of power in all directions, i.e., it has a directivity of 1 (0 dB) and a gain of 1 (0 dB) if the efficiency were 100%.

A three dimensional array with an arbitrary geometry is shown in Fig. 1.2. To simplify throughout this discussion, it is assumed that the source of the wave is in the far field of the array, and the incident wave can be treated as a plane wave. In spherical coordinates, the vector from the origin to the n -th element of the array is given by $\vec{r}_m = (\rho_m, \theta, \phi_m)$, and $-\hat{k} = (1, \theta, \phi)$ is the vector in the direction of the source of an incident wave. Throughout this discussion, it is assumed that the source of the wave is in the far field of the array and the incident wave can be treated as a plane wave. To find the array factor, it is necessary to find the relative phase of the received plane wave at each element. The phase is referred to as the phase of the plane wave at the origin. Thus, the phase of the received plane wave at the n -th element is the phase constant $\beta = \frac{2\pi}{\lambda}$ multiplied by the projection of the element position \vec{r}_m on to the plane wave arrival vector $-\hat{k}$. This is given by $-\vec{k} \bullet \vec{r}_m$ with the dot product taken in rectangular coordinates.

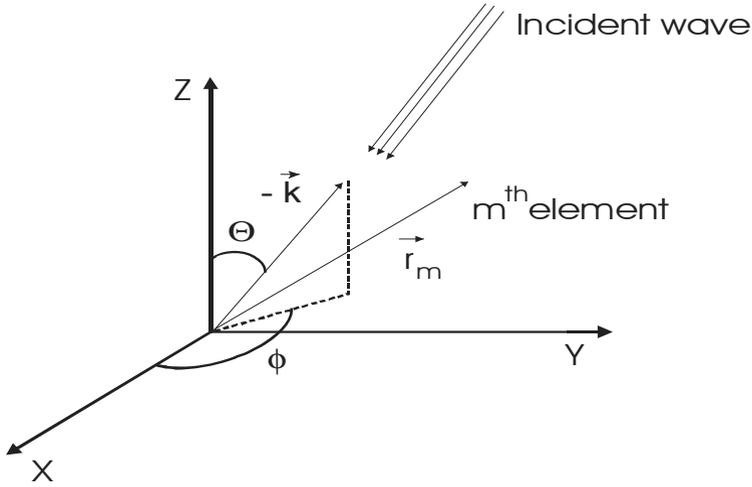


Fig. 1.2 A three dimensional array with an arbitrary geometry

In rectangular coordinates, $-\hat{k} = \sin \theta \cos \phi \hat{x} + \sin \theta \sin \phi \hat{y} + \cos \theta \hat{z} = r^{-2}$ and $\vec{r}_m = \rho_m \sin \theta_m \cos \phi_m \hat{x} + \rho_m \sin \theta_m \sin \phi_m \hat{y} + \rho_m \cos \theta_m \hat{z}$, and the relative phase of the incident wave at the n -th element is:

$$\begin{aligned} \zeta_m &= -\vec{k} \cdot \vec{r}_m \\ &= \beta \rho_m (\sin \theta \cos \phi \sin \theta_m \cos \phi_m + \sin \theta \sin \phi \sin \theta_m \sin \phi_m + \cos \theta \cos \theta_m) \\ &= \beta (x_m \sin \theta \cos \phi + y_m \sin \theta \sin \phi + z_m \cos \theta). \end{aligned}$$

For an array of M elements, the array factor is given by [3].

$$AF(\theta, \phi) = \sum_{m=1}^M I_m e^{j(\zeta_m + \delta_m)},$$

where I_m is the magnitude and δ_m is the phase of the weighting of the m -th element. The normalized array factor is given by:

$$f(\theta, \phi) = \frac{AF(\theta, \phi)}{\max\{|AF(\theta, \phi)|\}}.$$

Note this would be the same as the array pattern if the array consisted of ideal isotropic elements.

As an example, let us consider an array of any 2 identical antennas. The power